Future Topper

Half-Yearly Examination Practice Paper 1

Class IX Mathematics (041)

Time allowed: 3 hours Maximum marks: 80

Syllabus covered: Ganita Manjari Part I, Chapters 1 to 8 (coordinates, linear polynomials, the world of numbers, algebraic identities, circles, perimeter and area, probability, sequences and progressions). Schools split the half-yearly syllabus in different ways, so check your school's list before you start.

General instructions

1. This paper has 38 questions in five sections, A to E. All questions are compulsory.
2. Section A: questions 1 to 20 carry 1 mark each (18 multiple-choice and 2 assertion-reason).
3. Section B: questions 21 to 25 carry 2 marks each.
4. Section C: questions 26 to 31 carry 3 marks each.
5. Section D: questions 32 to 35 carry 5 marks each.
6. Section E: questions 36 to 38 are case-based, with sub-parts of 1, 1 and 2 marks.
7. Internal choice is given in two questions each of Sections B, C and D, and in the 2-mark part of every case study.
8. Draw neat figures wherever needed. Take $\pi = \frac{22}{7}$ unless stated otherwise. Calculators are not allowed.

Section A

Questions 1 to 20 carry 1 mark each

1. The distance of the point $P(-3, 4)$ from the origin is **1**

(a) 3 units (b) 4 units
(c) 7 units (d) 5 units

2. The midpoint of the segment joining $A(2, -1)$ and $B(6, 5)$ is **1**

(a) $(4, 2)$ (b) $(2, 3)$
(c) $(8, 4)$ (d) $(4, 3)$

3. Which of the following is a linear polynomial? **1**

(a) $x^2 + 1$ (b) $3 - 2x$
(c) $\dfrac{1}{x} + 2$ (d) $\sqrt{x} + 1$

4. For the line $y = 5 - 3x$, the slope and the y-intercept are respectively **1**

(a) 5 and -3 (b) -3 and 5
(c) 3 and 5 (d) -3 and -5

5. Which of the following numbers is irrational? **1**

(a) $\sqrt{49}$ (b) $0.\overline{3}$
(c) $\sqrt{12}$ (d) $\dfrac{22}{7}$

6. The decimal expansion of $\dfrac{7}{8}$ is **1**

(a) 0.875, terminating (b) $0.8\overline{75}$, non-terminating recurring
(c) 0.78, terminating (d) non-terminating and non-recurring

7. The value of $|-7| - |3|$ is **1**

(a) -10 (b) -4
(c) 10 (d) 4

8. Using a suitable identity, 103×97 equals **1**

(a) 9991 (b) 10009
(c) 9919 (d) 9999

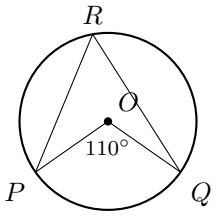
9. If $x + \frac{1}{x} = 4$, then $x^2 + \frac{1}{x^2}$ equals **1**

(a) 16 (b) 14
(c) 18 (d) 12

10. Which of the following is a factor of $x^2 - 5x + 6$? **1**

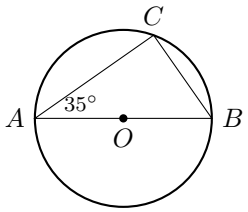
(a) $x + 2$ (b) $x + 3$
(c) $x - 6$ (d) $x - 3$

11. In the figure, O is the centre of the circle and $\angle POQ = 110°$. The value of $\angle PRQ$ is **1**



(a) $110°$ (b) $70°$
(c) $55°$ (d) $220°$

12. In the figure, AB is a diameter of the circle and $\angle CAB = 35°$. Then $\angle CBA$ is **1**



(a) $35°$ (b) $45°$
(c) $55°$ (d) $90°$

13. A chord of length 16 cm is drawn in a circle of radius 10 cm. Its distance from the centre is **1**

(a) 4 cm (b) 8 cm
(c) 12 cm (d) 6 cm

14. The length of the arc of a sector of radius 14 cm and angle $90°$ is **1**

(a) 11 cm (b) 22 cm
(c) 44 cm (d) 154 cm

15. The area of a triangle with sides 5 cm, 12 cm and 13 cm is **1**

(a) 30 cm^2 (b) 60 cm^2
(c) 65 cm^2 (d) 78 cm^2

16. A fair die is thrown once. The probability of getting a number greater than 4 is **1**

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$
(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

17. Which of the following cannot be the probability of an event? **1**

(a) 0 (b) 0.7
(c) 1.2 (d) $\frac{3}{5}$

18. The 10th term of the sequence 3, 7, 11, 15, ... is **1**

(a) 37 (b) 39

(c) 43 (d) 40

19. **Assertion (A):** The number 0.101001000100001... is irrational. **1**
Reason (R): A number whose decimal expansion is non-terminating and non-recurring is irrational.

(a) Both A and R are true, and R is the correct explanation of A.

(b) Both A and R are true, but R is not the correct explanation of A.

(c) A is true, but R is false.

(d) A is false, but R is true.

20. **Assertion (A):** The sequence 2, 6, 18, 54, ... is an arithmetic progression. **1**
Reason (R): In a geometric progression, each term after the first is obtained by multiplying the previous term by a fixed non-zero number.

(a) Both A and R are true, and R is the correct explanation of A.

(b) Both A and R are true, but R is not the correct explanation of A.

(c) A is true, but R is false.

(d) A is false, but R is true.

Section B

Questions 21 to 25 carry 2 marks each

21. Find two rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$. **2**

22. Using the distance formula, show that the points $A(1,1)$, $B(4,5)$ and $C(7,9)$ are collinear. **2**

OR

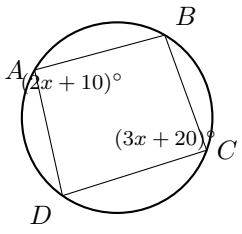
Find the point on the x-axis that is equidistant from $(2,3)$ and $(6,-1)$.

23. Factorise $4x^2 - 12x + 9$ using a suitable identity. **2**

OR

Simplify $\dfrac{x^2-9}{x^2+5x+6}$.

24. In the figure, $ABCD$ is a cyclic quadrilateral with $\angle A = (2x+10)^\circ$ and $\angle C = (3x+20)^\circ$. Find x, and hence find $\angle A$ and $\angle C$. **2**



25. A bag has 5 red, 3 blue and 2 green balls. One ball is taken out at random. Find the probability that the ball is (i) blue, (ii) not red. **2**

Section C

Questions 26 to 31 carry 3 marks each

26. Prove that $\sqrt{3}$ is irrational. **3**

27. A taxi charges a fixed amount of ₹50 and then ₹15 for every kilometre travelled. **3**

(i) Write the fare y (in ₹) for a journey of x km as a linear polynomial.

(ii) Find the fare for a 12 km journey.

(iii) What do the slope and the y-intercept of this relation stand for?

OR

A candle 30 cm long burns down by 2 cm every hour.

(i) Write its length L (in cm) after t hours as a linear polynomial.

(ii) After how many hours will the candle be 12 cm long?

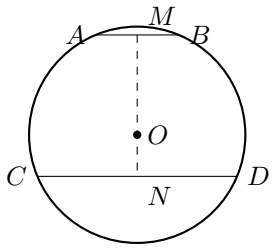
(iii) Is this linear growth or linear decay? Give a reason.

28. (i) If $a + b + c = 10$ and $ab + bc + ca = 31$, find $a^2 + b^2 + c^2$. **3**
(ii) Factorise $9x^2 - 16y^2$.

29. The 3rd term of an arithmetic progression is 12 and its 7th term is 28. Find the first term, the common difference and the 20th term. **3**

30. A triangular park has sides 26 m, 28 m and 30 m. Find its area using Heron's formula. Also find the cost of laying grass on it at ₹20 per m^2. **3**

31. In the figure, AB and CD are parallel chords of a circle with centre O and radius 13 cm, on opposite sides of the centre. $AB = 10$ cm and $CD = 24$ cm. Find the distance between the chords. **3**



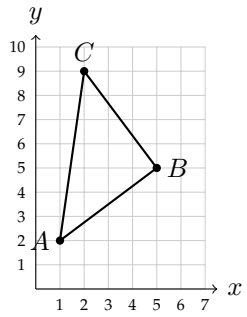
OR

Prove that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

Section D

Questions 32 to 35 carry 5 marks each

32. The figure shows triangle ABC with vertices $A(1, 2)$, $B(5, 5)$ and $C(2, 9)$ on a coordinate grid. **5**



(i) Find the lengths of AB, BC and AC.

(ii) Show that triangle ABC is a right-angled isosceles triangle, and name the right angle.

(iii) Find the midpoint M of AC.

(iv) Show that M is equidistant from A, B and C. What does this tell you about the circle through A, B and C?

33. (i) Factorise $x^2 + 7x + 12$. **5**
(ii) Factorise $6x^2 + 5x - 6$.
(iii) Show that $(a + b)^2 - (a - b)^2 = 4ab$. Hence find ab when $a + b = 9$ and $a - b = 5$.

34. A ball is dropped from a height of 81 m. After each bounce it rises to $\frac{2}{3}$ of the height from which it fell. **5**

(i) Write the heights reached after the first four bounces.

(ii) What kind of sequence do these heights form? State its first term and common ratio.

(iii) Write a rule for the height reached after the nth bounce.

(iv) After which bounce does the height first fall below 10 m?

OR

(i) Show that the sum of the first n natural numbers is $\dfrac{n(n+1)}{2}$, and use it to find $1 + 2 + 3 + \cdots + 100$.
(ii) A hall has 15 rows of seats. The first row has 20 seats and each row after it has 2 more seats than the row in front. How many seats are in the last row, and how many seats are there in all?

35. A flower bed is in the shape of a sector of a circle of radius 21 m with angle 120°. Find **5**

(i) the length of its arc,

(ii) its perimeter,

(iii) its area,

(iv) the cost of fencing it all round at ₹25 per metre.

OR

(i) Using Brahmagupta's formula, find the area of a cyclic quadrilateral with sides 7 cm, 15 cm, 20 cm and 24 cm.
(ii) Show that Heron's formula for a triangle is a special case of Brahmagupta's formula.

Section E

Questions 36 to 38 are case-based and carry 4 marks each

36. **How do students reach school?** A school surveyed 200 of its students about how they travel to school. The results were: bus 70, walking 40, bicycle 50, car 40. A student is chosen at random from this group. **4**

(i) Find the probability that the student comes by bus. (1)

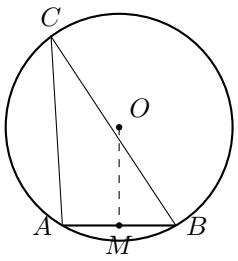
(ii) Find the probability that the student comes by car. (1)

(iii) Find the probability that the student uses a vehicle (bus, bicycle or car), and check that it adds up to 1 with the probability of walking. (2)

OR

The school has 1,200 students in all. Estimate how many of them come by bicycle, and find the probability that a student comes by car or bus. (2)

37. **A walkway in the garden.** A circular garden has centre O and radius 10 m. A straight walkway AB, also 10 m long, is laid across it, with a lamp post at its midpoint M. From a bench at C on the major arc, the walkway subtends $\angle ACB = 30^\circ$. **4**



(i) How far is the lamp post M from the centre O? (1)

(ii) Find $\angle AOB$. (1)

(iii) D is a point on the minor arc AB. Find $\angle ADB$. (2)

OR

A second walkway, also 10 m long, is laid elsewhere in the garden. How far is it from the centre, and what angle does it subtend at the centre? Give reasons. (2)

38. **Rahul's savings.** Rahul has ₹500 in his piggy bank. At the end of every month he adds ₹150. **4**

(i) Write the amounts in the piggy bank at the end of the first four months. What kind of sequence is this? (1)

(ii) Write the amount after n months as a linear polynomial in n. (1)

(iii) After how many months will the piggy bank hold ₹2,300? (2)

OR

His sister starts with ₹200 and adds ₹200 every month. After how many months will both have the same amount, and what will that amount be? (2)

***** End of question paper *****

Solutions

Marks for each step are not shown. Any correct method should get full credit.

Section A

1. $OP = \sqrt{(-3)^2 + 4^2} = \sqrt{25} = 5$. **Answer: (d) 5 units**

2. $M = \left(\frac{2+6}{2}, \frac{-1+5}{2}\right) = (4, 2)$. **Answer: (a)**

3. A linear polynomial has degree 1. $\frac{1}{x}$ and $\sqrt{x}$ are not polynomial terms, and $x^2 + 1$ has degree 2. **Answer: (b)** $3 - 2x$

4. Write it as $y = -3x + 5$. Slope $= -3$, y-intercept $= 5$. **Answer: (b)**

5. $\sqrt{49} = 7$, $0.\overline{3} = \frac{1}{3}$ and $\frac{22}{7}$ are rational. $\sqrt{12} = 2\sqrt{3}$ is irrational. **Answer: (c)**

6. $\frac{7}{8} = 0.875$. The denominator is 2^3, so the expansion terminates. **Answer: (a)**

7. $|-7| - |3| = 7 - 3 = 4$. **Answer: (d)**

8. $103 \times 97 = (100 + 3)(100 - 3) = 100^2 - 3^2 = 10000 - 9 = 9991$. **Answer: (a)**

9. $\left(x + \frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2}$, so $16 = x^2 + \frac{1}{x^2} + 2$, giving $x^2 + \frac{1}{x^2} = 14$. **Answer: (b)**

10. $x^2 - 5x + 6 = (x - 2)(x - 3)$. **Answer: (d)** $x - 3$

11. The angle at the circumference is half the angle at the centre: $\frac{110^\circ}{2} = 55^\circ$. **Answer: (c)**

12. The angle in a semicircle is 90°, so $\angle ACB = 90^\circ$ and $\angle CBA = 180^\circ - 90^\circ - 35^\circ = 55^\circ$. **Answer: (c)**

13. The perpendicular from the centre bisects the chord, so half the chord is 8 cm. Distance $= \sqrt{10^2 - 8^2} =$ 6 cm. **Answer: (d)**

14. Arc length $= \frac{90}{360} \times 2 \times \frac{22}{7} \times 14 = 22$ cm. **Answer: (b)**

15. $s = \frac{5+12+13}{2} = 15$. Area $= \sqrt{15 \times 10 \times 3 \times 2} = \sqrt{900} = 30$ cm^2. **Answer: (a)**

16. Favourable outcomes are 5 and 6, so $P = \frac{2}{6} = \frac{1}{3}$. **Answer: (d)**

17. A probability always lies between 0 and 1. **Answer: (c) 1.2**

18. $a = 3$, $d = 4$. $a_{10} = 3 + 9 \times 4 = 39$. **Answer: (b)**

19. The pattern of zeros keeps growing, so the expansion never ends and never repeats. Both statements are true and R explains A. **Answer: (a)**

20. $6 - 2 = 4$ but $18 - 6 = 12$, so the differences are not equal and A is false. Each term is 3 times the previous one, so the sequence is a GP, and R is a true statement. **Answer: (d)**

Section B

21. $\frac{3}{5} = \frac{12}{20}$ and $\frac{4}{5} = \frac{16}{20}$. Two rational numbers between them are $\frac{13}{20}$ and $\frac{14}{20} = \frac{7}{10}$. (Other correct answers, such as $\frac{15}{20} = \frac{3}{4}$, also get full marks.)

22. $AB = \sqrt{3^2+4^2} = 5$, $BC = \sqrt{3^2+4^2} = 5$, $AC = \sqrt{6^2+8^2} = 10$. Since $AB + BC = AC$, the three points lie on one line.
OR: Let the point be $P(x, 0)$. Then $PA^2 = PB^2$:
$(x-2)^2 + 9 = (x-6)^2 + 1 \Rightarrow x^2 - 4x + 13 = x^2 - 12x + 37 \Rightarrow 8x = 24 \Rightarrow x = 3$.
The point is $(3, 0)$. Check: $PA^2 = 1 + 9 = 10$ and $PB^2 = 9 + 1 = 10$.

23. $4x^2 - 12x + 9 = (2x)^2 - 2(2x)(3) + 3^2 = (2x-3)^2$.
OR: $\dfrac{x^2-9}{x^2+5x+6} = \dfrac{(x-3)(x+3)}{(x+2)(x+3)} = \dfrac{x-3}{x+2}$, for $x \neq -2, -3$.

24. Opposite angles of a cyclic quadrilateral add up to $180°$:
$(2x + 10) + (3x + 20) = 180 \Rightarrow 5x = 150 \Rightarrow x = 30$.
$\angle A = 70°$ and $\angle C = 110°$.

25. Total balls $= 10$. (i) $P(\text{blue}) = \frac{3}{10}$. (ii) Balls that are not red $= 5$, so $P(\text{not red}) = \frac{5}{10} = \frac{1}{2}$.

Section C

26. Suppose $\sqrt{3}$ is rational. Then $\sqrt{3} = \frac{p}{q}$, where p and q are integers, $q \neq 0$, and p and q have no common factor other than 1.
Squaring, $3q^2 = p^2$. So p^2 is a multiple of 3, which means p is a multiple of 3. Let $p = 3m$.
Then $3q^2 = 9m^2$, so $q^2 = 3m^2$. So q^2, and therefore q, is a multiple of 3.
Now 3 divides both p and q, which contradicts our assumption. Hence $\sqrt{3}$ is irrational.

27. (i) $y = 15x + 50$. (ii) $y = 15 \times 12 + 50 = 230$, so the fare is ₹230. (iii) The slope, 15, is the charge per kilometre. The y-intercept, 50, is the fixed charge you pay even before the taxi moves.
OR: (i) $L = 30 - 2t$. (ii) $30 - 2t = 12 \Rightarrow t = 9$ hours. (iii) Linear decay, because the length goes down by the same amount (2 cm) each hour. The slope, -2, is negative.

28. (i) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$, so $100 = a^2 + b^2 + c^2 + 62$ and $a^2 + b^2 + c^2 = 38$.
(ii) $9x^2 - 16y^2 = (3x)^2 - (4y)^2 = (3x - 4y)(3x + 4y)$.

29. $a + 2d = 12$ and $a + 6d = 28$. Subtracting, $4d = 16$, so $d = 4$ and $a = 4$.
$a_{20} = a + 19d = 4 + 76 = 80$.
First term 4, common difference 4, 20th term 80.

30. $s = \frac{26+28+30}{2} = 42$ m.
Area $= \sqrt{42 \times 16 \times 14 \times 12} = \sqrt{112896} = 336$ m^2.
Cost $= 336 \times 20 =$ ₹6,720.

31. Let $OM \perp AB$ and $ON \perp CD$. The perpendicular from the centre bisects a chord, so $AM = 5$ cm and $CN = 12$ cm.
$OM = \sqrt{13^2 - 5^2} = 12$ cm and $ON = \sqrt{13^2 - 12^2} = 5$ cm.
The chords are on opposite sides of the centre, so the distance between them is $12 + 5 = 17$ cm.
OR: Let AB be a chord of a circle with centre O, and let $OM \perp AB$, with M on AB. Join OA and OB. In right triangles OMA and OMB, by Pythagoras' theorem, $AM^2 = OA^2 - OM^2$ and $BM^2 = OB^2 - OM^2$.
$OA = OB$ (radii), so $AM^2 = BM^2$, which gives $AM = BM$. Hence OM bisects AB.

Section D

32. (i) $AB = \sqrt{4^2+3^2} = 5$, $BC = \sqrt{(-3)^2+4^2} = 5$, $AC = \sqrt{1^2+7^2} = \sqrt{50} = 5\sqrt{2}$.
(ii) $AB = BC$, so the triangle is isosceles. Also $AB^2 + BC^2 = 25+25 = 50 = AC^2$, so by the converse of Pythagoras' theorem, $\angle B = 90°$.
(iii) $M = \left(\frac{1+2}{2}, \frac{2+9}{2}\right) = \left(\frac{3}{2}, \frac{11}{2}\right)$.
(iv) $MA^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{7}{2}\right)^2 = \frac{50}{4}$, $MC^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{7}{2}\right)^2 = \frac{50}{4}$, $MB^2 = \left(\frac{7}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{50}{4}$.
So $MA = MB = MC = \frac{5\sqrt{2}}{2}$. M is the centre of the circle through A, B and C, and AC is its diameter. This fits the fact that the angle in a semicircle is a right angle.

33. (i) $x^2 + 7x + 12 = x^2 + 3x + 4x + 12 = (x+3)(x+4)$.
(ii) $6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6 = 3x(2x+3) - 2(2x+3) = (2x+3)(3x-2)$.
(iii) $(a+b)^2 - (a-b)^2 = (a^2+2ab+b^2) - (a^2-2ab+b^2) = 4ab$.
So $4ab = 9^2 - 5^2 = 56$, and $ab = 14$.

34. (i) $81 \times \frac{2}{3} = 54$, then 36, 24 and 16 m.
(ii) A geometric progression with first term 54 and common ratio $\frac{2}{3}$.
(iii) $h_n = 81 \times \left(\frac{2}{3}\right)^n$ m.
(iv) $h_4 = 16$, $h_5 = \frac{32}{3} \approx 10.67$, $h_6 = \frac{64}{9} \approx 7.11$. The height first falls below 10 m after the 6th bounce.
OR: (i) Let $S = 1 + 2 + \cdots + n$. Writing the sum backwards, $S = n + (n-1) + \cdots + 1$. Adding the two, each of the n pairs adds up to $n+1$, so $2S = n(n+1)$ and $S = \frac{n(n+1)}{2}$.
$1 + 2 + \cdots + 100 = \frac{100 \times 101}{2} = 5050$.
(ii) The rows form an AP: 20, 22, 24, ... Last row $= 20 + 14 \times 2 = 48$ seats.
Total $= 15 \times 20 + 2(1 + 2 + \cdots + 14) = 300 + 2 \times 105 = 510$ seats.

35. (i) Arc $= \frac{120}{360} \times 2 \times \frac{22}{7} \times 21 = 44$ m.
(ii) Perimeter $= 44 + 21 + 21 = 86$ m.
(iii) Area $= \frac{120}{360} \times \frac{22}{7} \times 21 \times 21 = 462$ m^2.
(iv) Cost $= 86 \times 25 =$ ₹2,150.
OR: (i) $s = \frac{7+15+20+24}{2} = 33$.
Area $= \sqrt{(s-a)(s-b)(s-c)(s-d)} = \sqrt{26 \times 18 \times 13 \times 9} = \sqrt{54756} = 234$ cm^2.
(ii) A triangle can be thought of as a cyclic quadrilateral with one side shrunk to zero. Put $d = 0$. Then $s = \frac{a+b+c}{2}$ and $s - d = s$, so the formula becomes $\sqrt{s(s-a)(s-b)(s-c)}$, which is Heron's formula.

Section E

36. (i) $\frac{70}{200} = \frac{7}{20}$. (ii) $\frac{40}{200} = \frac{1}{5}$.
(iii) Vehicle users $= 70 + 50 + 40 = 160$, so $P = \frac{160}{200} = \frac{4}{5}$. $P(\text{walk}) = \frac{40}{200} = \frac{1}{5}$, and $\frac{4}{5} + \frac{1}{5} = 1$.
OR: $P(\text{bicycle}) = \frac{50}{200} = \frac{1}{4}$, so about $\frac{1}{4} \times 1200 = 300$ students come by bicycle. $P(\text{car or bus}) = \frac{40+70}{200} = \frac{11}{20}$.

37. (i) M is the midpoint of chord AB, so $OM \perp AB$ and $AM = 5$ m. $OM = \sqrt{10^2 - 5^2} = \sqrt{75} = 5\sqrt{3} \approx 8.66$ m.
(ii) $\angle AOB = 2\angle ACB = 60°$. (This fits: $OA = OB = AB = 10$ m, so triangle OAB is equilateral.)
(iii) $ACBD$ is a cyclic quadrilateral, so $\angle ADB = 180° - 30° = 150°$.
OR: Equal chords of a circle are equidistant from the centre, so the second walkway is also $5\sqrt{3}$ m from O. Equal chords also subtend equal angles at the centre, so it subtends $60°$.

38. (i) ₹650, ₹800, ₹950, ₹1,100. This is an arithmetic progression with common difference 150.
(ii) Amount $= 500 + 150n$.
(iii) $500 + 150n = 2300 \Rightarrow 150n = 1800 \Rightarrow n = 12$ months.
OR: $500 + 150n = 200 + 200n \Rightarrow 50n = 300 \Rightarrow n = 6$. After 6 months both will have $500 + 900 =$ ₹1,400.