Future Topper

Annual Examination Practice Paper 2

Class IX Mathematics (041)

Time allowed: 3 hours Maximum marks: 80

Syllabus covered: the full current CBSE Class IX Mathematics curriculum. Unit weightage follows CBSE: Number System 7, Algebra 20, Coordinate Geometry 4, Geometry 25, Mensuration 14, Statistics and Probability 10.

General instructions

1. This paper has 38 questions in five sections, A to E. All questions are compulsory.
2. Section A: questions 1 to 20 carry 1 mark each (18 multiple-choice and 2 assertion-reason).
3. Section B: questions 21 to 25 carry 2 marks each.
4. Section C: questions 26 to 31 carry 3 marks each.
5. Section D: questions 32 to 35 carry 5 marks each.
6. Section E: questions 36 to 38 are case-based, with sub-parts of 1, 1 and 2 marks.
7. Internal choice is given in two questions each of Sections B, C and D, and in the 2-mark part of every case study.
8. Draw neat figures wherever needed. Take $\pi = \frac{22}{7}$ unless stated otherwise. Calculators are not allowed.

Section A

Questions 1 to 20 carry 1 mark each

1. $\sqrt{8} \times \sqrt{2}$ is equal to **1**

(a) 4 (b) $\sqrt{10}$

(c) 16 (d) $2\sqrt{2}$

2. The decimal expansion of $\frac{1}{11}$ is **1**

(a) $0.\overline{1}$ (b) $0.\overline{09}$

(c) 0.11 (d) 0.909

3. If $p(x) = x^2 - 3x + 2$, then $p(-1)$ is **1**

(a) 0 (b) 2

(c) 6 (d) -2

4. The graph of $y = 3$ is a line **1**

(a) parallel to the y-axis (b) passing through the origin

(c) along the x-axis (d) parallel to the x-axis

5. When $x = 2y$ is written as $ax + by + c = 0$, the values of a, b and c are **1**

(a) $1, -2, 0$ (b) $1, 2, 0$

(c) $2, -1, 0$ (d) $-1, -2, 0$

6. The common difference of the arithmetic progression $15, 11, 7, 3, \ldots$ is **1**

(a) 4 (b) -4

(c) 3 (d) -3

7. The sum of the first 20 natural numbers is **1**

(a) 200 (b) 190

(c) 210 (d) 420

8. The factorised form of $x^2 - \frac{1}{4}$ is **1**

(a) $\left(x - \frac{1}{4}\right)^2$ (b) $\left(x - \frac{1}{2}\right)^2$

(c) $\left(x - \frac{1}{4}\right)\left(x + \frac{1}{4}\right)$ (d) $\left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right)$

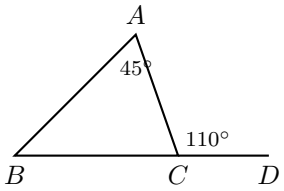
9. The distance between the points $(2, 3)$ and $(2, -5)$ is **1**

(a) 8 units (b) 2 units
(c) 7 units (d) 10 units

10. An angle is equal to its supplement. The angle is **1**

(a) $45°$ (b) $90°$
(c) $180°$ (d) $60°$

11. In the figure, side BC of triangle ABC is extended to D. If $\angle ACD = 110°$ and $\angle A = 45°$, then $\angle B$ is **1**



(a) $55°$ (b) $45°$
(c) $65°$ (d) $70°$

12. In triangle PQR, $PQ = PR$ and $\angle Q = 50°$. Then $\angle P$ is **1**

(a) $50°$ (b) $65°$
(c) $100°$ (d) $80°$

13. The diagonals of a rhombus **1**

(a) bisect each other at right angles (b) are always equal
(c) do not bisect each other (d) are always parallel

14. The medians of a triangle meet at a point that divides each median, from the vertex, in the ratio **1**

(a) $1 : 1$ (b) $2 : 1$
(c) $3 : 1$ (d) $1 : 2$

15. The longest chord of a circle of radius 5 cm has length **1**

(a) 5 cm (b) 25 cm
(c) 10 cm (d) 15 cm

16. The angle that a diameter subtends at any point on the circle is **1**

(a) $180°$ (b) $60°$
(c) $45°$ (d) $90°$

17. The volume of a sphere of radius 3 cm is **1**

(a) 36π cm^3 (b) 27π cm^3
(c) 12π cm^3 (d) 108π cm^3

18. A spinner has 8 equal sectors numbered 1 to 8. The probability that it stops on a prime number is **1**

(a) $\frac{3}{8}$ (b) $\frac{1}{2}$
(c) $\frac{5}{8}$ (d) $\frac{1}{4}$

19. **Assertion (A):** The point $(-3, -4)$ lies in the third quadrant. **1**
Reason (R): In the third quadrant, both coordinates of a point are negative.

(a) Both A and R are true, and R is the correct explanation of A.
(b) Both A and R are true, but R is not the correct explanation of A.
(c) A is true, but R is false.
(d) A is false, but R is true.

20. **Assertion (A):** $2 + \sqrt{3}$ is irrational. **1**
Reason (R): The sum of two irrational numbers is always irrational.

(a) Both A and R are true, and R is the correct explanation of A.
(b) Both A and R are true, but R is not the correct explanation of A.
(c) A is true, but R is false.
(d) A is false, but R is true.

Section B

Questions 21 to 25 carry 2 marks each

21. If $\dfrac{\sqrt{3}-1}{\sqrt{3}+1} = a + b\sqrt{3}$, find the rational numbers a and b. **2**

OR

Show that $1.\overline{27}$ is a rational number by writing it in the form $\frac{p}{q}$.

22. Write $2x + 3y = 12$ in the form $y = mx + c$. State its slope and y-intercept. **2**

OR

Write the equation of the line through the origin with slope 3. Does the point $(2, 6)$ lie on it?

23. Point C lies between A and B on a line segment, and $AC = BC$. Prove that $AC = \frac{1}{2}AB$. Name the Euclid's axiom you use. **2**

24. The mean of five numbers is 18. When one number is removed, the mean of the remaining four is 16. Find the number that was removed. **2**

OR

Find the median and the mode of 4, 7, 3, 7, 9, 2, 7, 5.

25. The midpoint of the segment joining $(a, 4)$ and $(6, b)$ is $(3, 5)$. Find a and b. **2**

Section C

Questions 26 to 31 carry 3 marks each

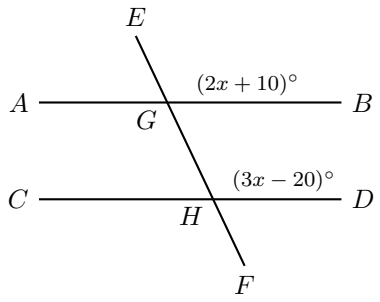
26. Find three irrational numbers between 2 and 3. Explain why each of them is irrational and lies between 2 and 3. **3**

27. If $x - \frac{1}{x} = 3$, find (i) $x^2 + \frac{1}{x^2}$ and (ii) $x^4 + \frac{1}{x^4}$. **3**

OR

Factorise $2x^2 - 7x + 3$. Check your answer by multiplying the factors.

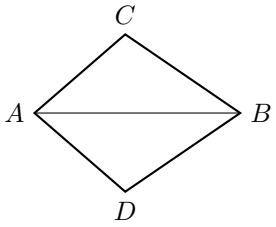
28. AB and CD are parallel lines. A transversal EF cuts AB at G and CD at H, with E above AB and F below CD. If $\angle EGB = (2x + 10)^\circ$ and $\angle GHD = (3x - 20)^\circ$, find x. Hence find $\angle AGH$ and $\angle GHC$. **3**



OR

Prove that when two lines intersect, the vertically opposite angles are equal.

29. In quadrilateral $ACBD$ shown, $AC = AD$ and AB bisects $\angle CAD$. Show that $\triangle ABC \cong \triangle ABD$. What can you say about BC and BD? **3**



30. A bag has 4 red and 6 white balls. A ball is drawn, its colour noted and the ball put back. A second ball is then drawn. **3**

(i) Draw a tree diagram for the two draws.

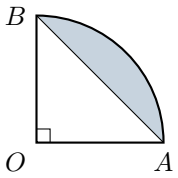
(ii) Find the probability that both balls are red.

(iii) Find the probability that one ball is red and the other is white.

31. A wheel has radius 35 cm. Find its circumference, and the number of complete turns it makes in covering 1.1 km. **3**

OR

In the figure, OAB is a sector of a circle with centre O, radius 14 cm and $\angle AOB = 90°$. Find the area of the shaded segment.



Section D

Questions 32 to 35 carry 5 marks each

32. Squares are made in a row using matchsticks. One square needs 4 matchsticks, two squares need 7, three need 10 and four need 13. **5**

(i) Is this an arithmetic progression? Give a reason.

(ii) Write a rule for the number of matchsticks needed for n squares.

(iii) How many matchsticks are needed for 25 squares?

(iv) What is the largest number of squares that can be made with 100 matchsticks?

OR

A culture has 500 bacteria. The number doubles every hour.

(i) Write the number of bacteria after 1, 2 and 3 hours.

(ii) Write a recursive rule and an explicit rule for the number after n hours.

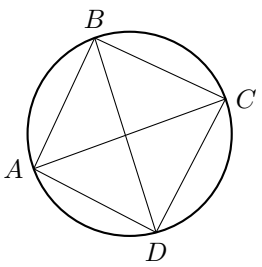
(iii) After how many complete hours will the number first go above 1,00,000?

33. (i) Prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and half of it. **5**
(ii) The sides of a triangle are 8 cm, 10 cm and 12 cm. Find the perimeter of the triangle formed by joining the midpoints of its sides.

OR

(i) Prove that a diagonal of a parallelogram divides it into two congruent triangles.
(ii) Hence show that the opposite sides of a parallelogram are equal.

34. (i) Prove that the angle subtended by an arc at the centre of a circle is double the angle it subtends at any point on the remaining part of the circle. **5**
(ii) In the figure, $ABCD$ is a cyclic quadrilateral whose diagonals meet at E. If $\angle DBC = 55°$ and $\angle BAC = 45°$, find $\angle BCD$.



35. A water tank in the shape of a cuboid is 2 m long, 1.5 m wide and 1 m deep, and has no lid. **5**

(i) Find how many litres it holds when full.

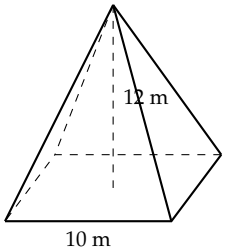
(ii) Find the area of metal sheet needed to make it.

(iii) A pipe fills it at 25 litres per minute. How long will it take to fill the empty tank?

(iv) A cylindrical tank of radius 0.7 m and height 2 m is also available. Which tank holds more water, and by how many litres?

OR

A pyramid has a square base of side 10 m and a height of 12 m, as shown.



(i) Find the slant height of each triangular face.

(ii) Find the total area of its four triangular faces.

(iii) Find its volume.

Section E

Questions 36 to 38 are case-based and carry 4 marks each

36. **The final grade.** A school works out the final grade as a weighted average: Term 1 counts for 30% and Term 2 for 70%. Meera scored 80 in Term 1 and 85 in Term 2. **4**

(i) Find the simple average of her two scores. (1)

(ii) Find her weighted average. (1)

(iii) With 80 in Term 1, what would she need in Term 2 to get a weighted average of 87? (2)

OR

Arjun scored 90 in Term 1 and 75 in Term 2. Whose weighted average is higher, and why does Arjun's higher Term 1 score not help him enough? (2)

37. **Choosing a data plan.** Plan A costs ₹100 a month plus ₹2 for every GB used. Plan B costs ₹180 a month plus ₹1 for every GB used. **4**

(i) Write the monthly cost y for x GB under each plan. (1)

(ii) Find the cost of 50 GB under Plan A. (1)

(iii) For how many GB do the two plans cost the same, and what is that cost? (2)

OR

Which plan is cheaper for 100 GB? What does the point where the two graphs meet tell you? (2)

38. **The park path.** A circular park has radius 21 m. A path 3.5 m wide runs all round it, just outside the park. **4**

(i) Find the circumference of the park. (1)

(ii) Find the outer radius and the outer circumference of the path. (1)

(iii) Find the area of the path. (2)

OR

The path is to be paved at ₹40 per m^2. Find the area of the path and the cost of paving. (2)

***** End of question paper *****

Solutions

Marks for each step are not shown. Any correct method should get full credit.

Section A

1. $\sqrt{8 \times 2} = \sqrt{16} = 4$. **Answer: (a)**

2. $1 \div 11 = 0.090909\cdots = 0.\overline{09}$. **Answer: (b)**

3. $(-1)^2 - 3(-1) + 2 = 1 + 3 + 2 = 6$. **Answer: (c)**

4. Every point on it has $y = 3$, so it runs parallel to the x-axis, 3 units above it. **Answer: (d)**

5. $x - 2y + 0 = 0$. **Answer: (a)**

6. $11 - 15 = -4$. **Answer: (b)**

7. $\frac{20 \times 21}{2} = 210$. **Answer: (c)**

8. $x^2 - \left(\frac{1}{2}\right)^2 = \left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right)$. **Answer: (d)**

9. $\sqrt{0^2 + (-8)^2} = 8$. **Answer: (a)**

10. $x + x = 180°$, so $x = 90°$. **Answer: (b)**

11. The exterior angle equals the sum of the interior opposite angles: $110° - 45° = 65°$. **Answer: (c)**

12. $\angle R = \angle Q = 50°$, so $\angle P = 180° - 100° = 80°$. **Answer: (d)**

13. **Answer: (a)**

14. The centroid divides each median in the ratio $2 : 1$. **Answer: (b)**

15. The longest chord is a diameter: $2 \times 5 = 10$ cm. **Answer: (c)**

16. The angle in a semicircle is a right angle. **Answer: (d)**

17. $\frac{4}{3}\pi(3)^3 = 36\pi$ cm^3. **Answer: (a)**

18. Primes from 1 to 8 are 2, 3, 5, 7. $P = \frac{4}{8} = \frac{1}{2}$. **Answer: (b)**

19. Both coordinates are negative, which is exactly what places the point in the third quadrant. **Answer: (a)**

20. A is true (a rational number plus an irrational number is irrational). R is false: for example, $\sqrt{3} + (-\sqrt{3}) = 0$, which is rational. **Answer: (c)**

Section B

21. $\dfrac{\sqrt{3}-1}{\sqrt{3}+1} \times \dfrac{\sqrt{3}-1}{\sqrt{3}-1} = \dfrac{3 - 2\sqrt{3} + 1}{3 - 1} = \dfrac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$. So $a = 2$ and $b = -1$.
OR: Let $x = 1.2727\ldots$. Then $100x = 127.2727\ldots$, so $99x = 126$ and $x = \frac{126}{99} = \frac{14}{11}$, which is rational.

22. $3y = -2x + 12$, so $y = -\frac{2}{3}x + 4$. Slope $= -\frac{2}{3}$, y-intercept $= 4$.
OR: $y = 3x$. When $x = 2$, $y = 6$, so $(2, 6)$ lies on the line.

23. $AC + BC = AB$, since together they make up the whole segment (things which coincide with one another are equal to one another).
Since $AC = BC$, $AC + AC = AB$, so $2AC = AB$ and $AC = \frac{1}{2}AB$.
Also used: things which are halves of the same thing are equal to one another.

24. Sum of five numbers $= 5 \times 18 = 90$. Sum of the remaining four $= 4 \times 16 = 64$. Removed number $= 90 - 64 = 26$.
OR: In order: 2, 3, 4, 5, 7, 7, 7, 9. Median $= \frac{5+7}{2} = 6$. Mode $= 7$.

25. $\frac{a+6}{2} = 3$ gives $a = 0$. $\frac{4+b}{2} = 5$ gives $b = 6$.

Section C

26. $2 = \sqrt{4}$ and $3 = \sqrt{9}$. So $\sqrt{5}$, $\sqrt{6}$ and $\sqrt{7}$ lie between 2 and 3.
None of 5, 6, 7 is a perfect square, so their square roots are irrational.
(Other answers, such as $2.101001000100001\ldots$, are also correct if properly justified.)

27. (i) $\left(x - \frac{1}{x}\right)^2 = x^2 - 2 + \frac{1}{x^2}$, so $9 = x^2 + \frac{1}{x^2} - 2$ and $x^2 + \frac{1}{x^2} = 11$.
(ii) $\left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + 2 + \frac{1}{x^4}$, so $121 = x^4 + \frac{1}{x^4} + 2$ and $x^4 + \frac{1}{x^4} = 119$.
OR: $2x^2 - 7x + 3 = 2x^2 - 6x - x + 3 = 2x(x-3) - 1(x-3) = (2x-1)(x-3)$.
Check: $(2x-1)(x-3) = 2x^2 - 6x - x + 3 = 2x^2 - 7x + 3$.

28. $\angle EGB$ and $\angle GHD$ are corresponding angles, so they are equal:
$2x + 10 = 3x - 20 \Rightarrow x = 30$. Each angle is $70°$.
$\angle AGH = \angle EGB = 70°$ (vertically opposite).
$\angle GHC = 180° - \angle GHD = 110°$ (linear pair).
OR: Let lines AB and CD meet at O. $\angle AOC + \angle AOD = 180°$ (linear pair) and $\angle AOD + \angle BOD = 180°$ (linear pair). So $\angle AOC + \angle AOD = \angle AOD + \angle BOD$, which gives $\angle AOC = \angle BOD$. In the same way, $\angle AOD = \angle BOC$.

29. In $\triangle ABC$ and $\triangle ABD$: $AC = AD$ (given), $\angle CAB = \angle DAB$ (AB bisects $\angle CAD$), and $AB = AB$ (common). So $\triangle ABC \cong \triangle ABD$ by SAS. Hence $BC = BD$ (corresponding parts of congruent triangles).

30. (i) First draw: R with probability $\frac{2}{5}$ and W with $\frac{3}{5}$. From each branch, the second draw again splits into R ($\frac{2}{5}$) and W ($\frac{3}{5}$), giving outcomes RR, RW, WR and WW.
(ii) $P(RR) = \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$.
(iii) $P(RW) + P(WR) = \frac{2}{5} \times \frac{3}{5} + \frac{3}{5} \times \frac{2}{5} = \frac{12}{25}$.

31. Circumference $= 2 \times \frac{22}{7} \times 35 = 220$ cm. 1.1 km $= 1,10,000$ cm. Number of turns $= \frac{110000}{220} = 500$.
OR: Area of sector $= \frac{90}{360} \times \frac{22}{7} \times 14 \times 14 = 154$ cm^2. Area of $\triangle OAB = \frac{1}{2} \times 14 \times 14 = 98$ cm^2. Area of segment $= 154 - 98 = 56$ cm^2.

Section D

32. (i) Yes. Each new square adds 3 matchsticks, so the difference between terms is always 3.
(ii) $a_n = 4 + (n-1) \times 3 = 3n + 1$.
(iii) $3(25) + 1 = 76$ matchsticks.
(iv) $3n + 1 \leq 100$ gives $n \leq 33$. So 33 squares, which use exactly 100 matchsticks.
OR: (i) 1,000, 2,000, 4,000.
(ii) Recursive: $B_n = 2B_{n-1}$, with $B_0 = 500$. Explicit: $B_n = 500 \times 2^n$.
(iii) We need $2^n > 200$. $2^7 = 128$ and $2^8 = 256$, so after 8 hours (1,28,000 bacteria).

33. (i) In $\triangle ABC$, let E and F be the midpoints of AB and AC. Extend EF to D so that $EF = FD$, and join CD.
In $\triangle AEF$ and $\triangle CDF$: $AF = CF$, $EF = DF$, and $\angle AFE = \angle CFD$ (vertically opposite). So the triangles are congruent (SAS).
Hence $CD = AE = BE$, and $\angle EAF = \angle DCF$, which are alternate angles, so $CD \parallel AB$, that is, $CD \parallel BE$.
So $EBCD$ has a pair of opposite sides equal and parallel, and is a parallelogram. Therefore $ED \parallel BC$, that is, $EF \parallel BC$, and $EF = \frac{1}{2}ED = \frac{1}{2}BC$.
(ii) Each side of the new triangle is half a side of the original: 4, 5 and 6 cm. Perimeter $= 15$ cm.
OR: (i) In parallelogram $ABCD$, join AC. In $\triangle ABC$ and $\triangle CDA$: $\angle BAC = \angle DCA$ (alternate angles, $AB \parallel DC$), $\angle BCA = \angle DAC$ (alternate angles, $BC \parallel AD$), and $AC = CA$ (common). So $\triangle ABC \cong \triangle CDA$ by ASA.
(ii) Corresponding parts give $AB = CD$ and $BC = DA$.

34. (i) Let arc PQ subtend $\angle POQ$ at the centre O and $\angle PAQ$ at a point A on the remaining part. Join AO and extend it to a point B.
In $\triangle AOP$, $OA = OP$, so $\angle OAP = \angle OPA$. The exterior angle $\angle POB = \angle OAP + \angle OPA = 2\angle OAP$.
In the same way, $\angle QOB = 2\angle OAQ$.
Adding: $\angle POQ = 2(\angle OAP + \angle OAQ) = 2\angle PAQ$.
(ii) $\angle CAD = \angle CBD = 55^\circ$ (angles in the same segment). So $\angle BAD = 45^\circ + 55^\circ = 100^\circ$.
Opposite angles of a cyclic quadrilateral add up to 180°, so $\angle BCD = 180^\circ - 100^\circ = 80^\circ$.

35. (i) Volume $= 2 \times 1.5 \times 1 = 3$ m$^3 = 3,000$ litres.
(ii) Base + four walls $= lb + 2(l + b)h = 3 + 2(3.5)(1) = 10$ m^2.
(iii) $\frac{3000}{25} = 120$ minutes, or 2 hours.
(iv) Cylinder: $\frac{22}{7} \times 0.7 \times 0.7 \times 2 = 3.08$ m$^3 = 3,080$ litres. The cylindrical tank holds 80 litres more.
OR: (i) Slant height $= \sqrt{12^2 + 5^2} = 13$ m, since the foot of the slant height is 5 m from the centre of the base.
(ii) Four faces $= 4 \times \frac{1}{2} \times 10 \times 13 = 260$ m^2.
(iii) Volume $= \frac{1}{3} \times 10 \times 10 \times 12 = 400$ m^3.

Section E

36. (i) $\frac{80+85}{2} = 82.5$. (ii) $0.3 \times 80 + 0.7 \times 85 = 24 + 59.5 = 83.5$.
(iii) $24 + 0.7x = 87$, so $0.7x = 63$ and $x = 90$.
OR: Arjun: $0.3 \times 90 + 0.7 \times 75 = 27 + 52.5 = 79.5$. Meera's 83.5 is higher. Term 2 carries 70% of the weight, so Arjun's lower Term 2 score pulls his grade down more than his Term 1 score lifts it.

37. (i) Plan A: $y = 100 + 2x$. Plan B: $y = 180 + x$. (ii) $100 + 2(50) =$ ₹200.
(iii) $100 + 2x = 180 + x$ gives $x = 80$ GB, and the cost is $100 + 160 =$ ₹260.
OR: At 100 GB, Plan A costs ₹300 and Plan B costs ₹280, so Plan B is cheaper. The graphs meet at $(80, 260)$: at 80 GB both cost ₹260. Below 80 GB Plan A is cheaper, and above it Plan B is cheaper.

38. (i) $2 \times \frac{22}{7} \times 21 = 132$ m.
(ii) Outer radius $= 21 + 3.5 = 24.5$ m. Outer circumference $= 2 \times \frac{22}{7} \times 24.5 = 154$ m.
(iii) Area $= \frac{22}{7}(24.5^2 - 21^2) = \frac{22}{7} \times 159.25 = 500.5$ m^2.
OR: Area $= 500.5$ m^2 (as above). Cost $= 500.5 \times 40 =$ ₹20,020.